

The gluon contents of the η and η' mesons

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Abstract. It is reported on a leading-twist analysis of the $\eta - \gamma$ and $\eta' - \gamma$ transition form factors. The analysis allows for an estimate of the lowest Gegenbauer coefficients of the quark and gluon distribution amplitudes.

One of the simplest exclusive observables is the form factor $F_{P\gamma^{(*)}}$ for the transitions from a real or virtual photon to a pseudoscalar meson P . Its behaviour at large momentum transfer is determined by the expansion of a product of two electromagnetic currents about light-like distances. The form factor then factorizes [1] into a hard scattering amplitude and a soft matrix element, parameterized by a process-independent meson distribution amplitude Φ_P . For space-like momentum transfer the form factor can be accessed in $e^+e^- \rightarrow e^+e^-P$. Such measurements have been carried through for quasi-real photons by CLEO [2] and L3 [3]. From the data on the form factors one may extract information about the meson distribution amplitudes by fitting the theoretical results to the experimental data. Here, in this talk, it is reported on recent attempts [4, 5] to perform such analyses to leading-twist NLO accuracy in the cases of the η and η' mesons.

As the valence Fock components of the η and η' mesons $SU(3)_F$ singlet and octet combinations of quark-antiquark parton states are chosen

$$|q\bar{q}_1\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}, \quad |q\bar{q}_8\rangle = |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6}. \quad (1)$$

In addition the two-gluon Fock state, $|gg\rangle$, is to be taken into account which also possesses flavour-singlet quantum numbers and contributes to leading-twist order. Associated to each valence Fock component of the meson P is a distribution amplitude denoted by Φ_{Pi} ($i = 1, 8$) and Φ_{Pg} . The distribution amplitudes possess Gegenbauer expansions [1]

$$\begin{aligned} \Phi_{Pi}(\xi, \mu_F) &= \frac{3}{2}(1 - \xi^2) \left[1 + \sum_{n=2,4,\dots} B_{Pn}^{(i)}(\mu_F) C_n^{3/2}(\xi) \right], \\ \Phi_{Pg}(\xi, \mu_F) &= \frac{1}{16}(1 - \xi^2)^2 \sum_{n=2,4,\dots} B_{Pn}^{(g)}(\mu_F) C_{n-1}^{5/2}(\xi), \end{aligned} \quad (2)$$

where $\xi = 2x - 1$, and x is the usual momentum fraction carried by the quark inside the meson. The Gegenbauer coefficients, B_{Pn} , which encode the soft physics, evolve with the factorization scale μ_F according to the relevant anomalous dimensions. The essential

point is that the singlet and gluon coefficients mix under evolution

$$B_{Pn}^{(1)}(\mu_F) \leftrightarrow B_{Pn}^{(g)}(\mu_F), \quad (3)$$

and that all coefficients evolve to zero for asymptotically large factorization scales. Hence

$$\Phi_{Pi} \rightarrow \Phi_{AS} = \frac{3}{2}(1 - \xi^2), \quad \Phi_{Pg} \rightarrow 0, \quad \text{for } \mu_F \rightarrow \infty. \quad (4)$$

It is important to note that the gluon distribution amplitude goes along with the following projector of a state of two incoming collinear gluons (colours a, b , Lorentz indices μ, ν and momentum fractions $x, 1-x$) onto a pseudoscalar meson state

$$\mathcal{P}_{\mu\nu,ab}^g = \frac{i}{2} \sqrt{\frac{C_F}{n_f}} \frac{\delta_{ab}}{\sqrt{N_c^2 - 1}} \frac{\varepsilon_{\perp\mu\nu}}{x(1-x)}. \quad (5)$$

The anomalous dimensions have to be normalized accordingly [5]. The components of the transverse polarization tensor are $\varepsilon_{\perp 12} = -\varepsilon_{\perp 21} = 1$ and zero for all others.

The $\gamma^*(q, \mu) \gamma^{(*)}(q', \nu) \rightarrow P(p)$ vertex is parameterized as

$$\Gamma^{\mu\nu} = ie_0^2 F_{P\gamma}(\bar{Q}, \omega) \varepsilon^{\mu\nu\alpha\beta} q_\alpha q'_\beta, \quad (6)$$

where $Q^2 = -q^2 \geq 0$, $Q'^2 = -q'^2 \geq 0$ and

$$\bar{Q}^2 = \frac{1}{2}(Q^2 + Q'^2), \quad \omega = \frac{Q^2 - Q'^2}{Q^2 + Q'^2}. \quad (7)$$

Due to Bose symmetry the transition form factor is symmetric in ω . To leading-twist NLO accuracy the transition form factor reads ($P = \eta, \eta'$)

$$\begin{aligned} F_{P\gamma^*} &= \frac{2}{3\sqrt{3}\bar{Q}^2} \int_{-1}^1 \frac{d\xi}{1-\xi^2\omega^2} \left\{ \left[\frac{f_P^{(8)}}{2\sqrt{2}} \Phi_{P8}(\xi, \mu_F) + f_P^{(1)} \Phi_{P1}(\xi, \mu_F) \right] \right. \\ &\times \left. \left[1 + \frac{\alpha_s(\mu_R)}{4\pi} \mathcal{K}_q(\omega, \xi, \bar{Q}^2) \right] + f_P^{(1)} \Phi_{Pg}(\xi, \mu_F) \frac{\alpha_s(\mu_R)}{4\pi} \mathcal{K}_g(\omega, \xi, \bar{Q}^2) \right\}. \quad (8) \end{aligned}$$

The NLO hard scattering kernels, \mathcal{K} , are calculated from the Feynman graphs shown in Fig. 1. The results - in the $\overline{\text{MS}}$ scheme - can be found in the literature, see for instance [4, 5, 6]. The decay constants, $f_P^{(i)}$, are defined by matrix elements of $SU(3)_F$ singlet and octet axial vector currents:

$$\langle 0 | J_{5\mu}^{(i)} | P(p) \rangle = i f_P^{(i)} p_\mu. \quad (9)$$

The singlet decay constant $f_P^{(1)}$ depends on the scale [7] but the anomalous dimension controlling it is of order α_s^2 . In a NLO calculation this effect is to be neglected for consistency. Note that the octet part of (8) also holds for the $\pi - \gamma$ form factor with the obvious replacement $\Phi_{P8} \rightarrow \Phi_\pi$, $f_P^{(8)} \rightarrow \sqrt{3}f_\pi$.

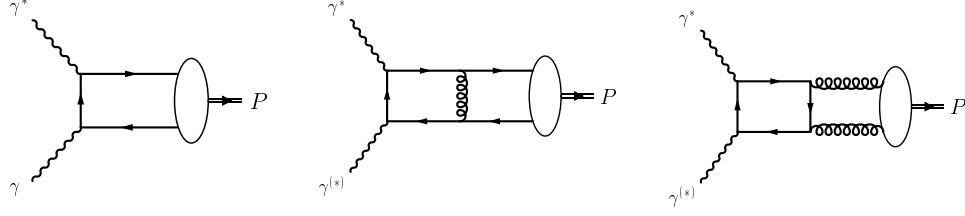


FIGURE 1. Sample Feynman graphs contributing to the transition form factors

Of particular interest is the limit $\omega \rightarrow 0$. Inserting the Gegenbauer expansion (2) into (8), one finds that the Gegenbauer coefficients of the quark and gluon distribution amplitudes first appears at order ω^n [4]. Hence, one obtains the prediction

$$F_{P\gamma^*}(\bar{Q}^2, \omega) = \frac{\sqrt{2}}{3\sqrt{3}} \frac{f_P^8 + 2\sqrt{2}f_P^1}{\bar{Q}^2} \left[1 - \frac{\alpha_s}{\pi} \right] + \mathcal{O}(\omega^2, \alpha_s^2). \quad (10)$$

Since the decay constants are known to amount to

$$f_\eta^{(8)} = 1.17f_\pi, \quad f_\eta^{(1)} = 0.19f_\pi, \quad f_{\eta'}^{(8)} = -0.46f_\pi, \quad f_{\eta'}^{(1)} = 1.15f_\pi, \quad (11)$$

with a accuracy of about 5% [8], (10) is a parameter-free prediction of QCD to leading-twist accuracy. Its theoretical status is comparable to that of the Bjorken sum rule [9]

$$\int_0^1 dx [g_1^p(x) - g_1^n(x)] = \frac{1}{6} \frac{G_A}{G_V} \left[1 - \frac{\alpha_s}{\pi} - 3.583 \left(\frac{\alpha_s}{\pi} \right)^2 - 20.215 \left(\frac{\alpha_s}{\pi} \right)^3 + \dots \right], \quad (12)$$

and a few other observables among which is the famous result for the cross section ratio of e^+e^- annihilation into hadrons and into a pair of muons. It is known [10] that the perturbative series of the transition form factors are identical to that of the Bjorken sum rule. The prediction (10) well deserves experimental verification but there is no data as yet.

The real photon case, $\omega = 1$, is another interesting limit. Here data is available [2, 3] from which information about the distribution amplitudes can be extracted. For the case of the pion such analyses have been carried through immediately after the advent of the CLEO data in Ref. [11, 12] and, recently, in much greater detail in [4]. The η and η' data have been analyzed within the modified perturbative approach in [13] and to leading twist NLO accuracy in [5]. Since the present quality of the data does not suffice to determine all six distribution amplitudes, one has to simplify matters and employ an $\eta - \eta'$ mixing scheme in order to reduce the number of free parameters. Since in hard processes only small spatial quark-antiquark separations are of relevance, it is sufficiently suggestive to embed the particle dependence and the mixing behaviour of the valence Fock components solely into the decay constants which play the role of wave functions at the origin. Following [8, 13], one may therefore take

$$\Phi_{Pi} = \Phi_i, \quad \Phi_{Pg} = \Phi_g. \quad (13)$$

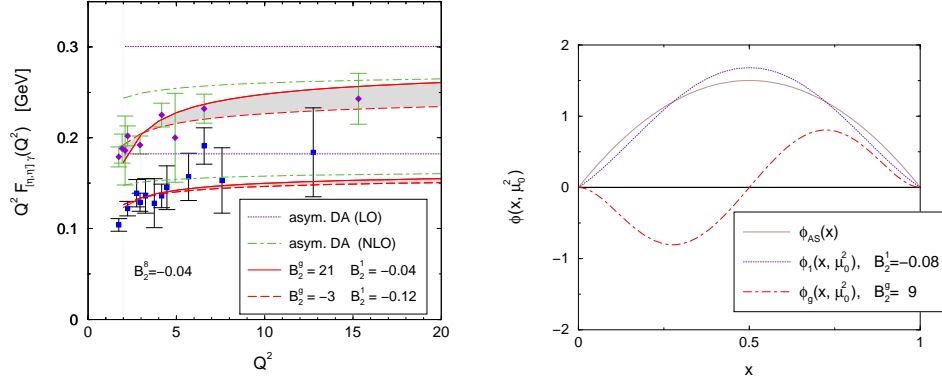


FIGURE 2. Left: The scaled $P\gamma$ transition form factor vs. Q^2 . Data taken from [2, 3]; rombs represent the η' data, squares the η ones. Right: The flavour-singlet and gluon distribution amplitudes at the scale $\mu_0 = 1 \text{ GeV}^2$

This assumption is further supported by the observations made in [13] that, as is the case for the pion [4, 11, 12] the quark distribution amplitudes are close to the asymptotic form, Φ_{AS} , for which the particle independence (13) holds trivially. The analysis is further simplified by truncating the Gegenbauer series in (2) at $n = 2$. The coefficients $B_2^{(i)}$, acting for all others, parameterize the deviations from the asymptotic form of the distribution amplitudes. Clearly, this is a serious assumption (note that to LO accuracy the transition form factors only fix the sum $1 + \sum B_n^{(i)}$) but in view of the large experimental errors as well of the limited range of momentum transfer in which data is available, one is forced to do so. Truncation at $n = 4$ does not lead to reliable results, all contributing Gegenbauer coefficients are highly correlated. A fit to the CLEO and L3 data provides

$$B_2^{(8)}(\mu_0) = -0.04 \pm 0.04, \quad B_2^{(1)}(\mu_0) = -0.08 \pm 0.04, \quad B_2^{(g)}(\mu_0) = 9 \pm 12, \quad (14)$$

where the following scales have been chosen: $\mu_0 = 1 \text{ GeV}$, $\mu_F = Q$, $\mu_R = Q/\sqrt{2}$. The use of $\mu_F = Q/\sqrt{2}$ instead leads to values of the Gegenbauer coefficients which agree with those quoted in (14) almost perfectly. For comparison, B_2^π takes a value of -0.06 ± 0.03 as determined in [4].

The fit is compared to the data in Fig. 2. The insensitivity of the $\eta - \gamma$ transition form factor to the gluonic distribution amplitude is clearly seen which comes about as a consequence of the smallness of $f_\eta^{(1)}$, see (11). Although the present data are compatible with a leading-twist analysis as Fig. 2, the existence of power and/or higher-twist corrections cannot be excluded. This is a source of theoretical uncertainties in the results (14). Thus, for instance, the use of the modified perturbative approach in which quark transverse degrees of freedom and Sudakov suppressions are taken into account, leads to good agreement with experiment for the asymptotic distribution amplitudes [13].

Within errors the quark Gegenbauer coefficients for the octet and singlet case agree with each other and with the pion one. This implies not only approximate flavour symmetry but also the approximate validity of the OZI rule which is a prerequisite of

the quark-flavour mixing scheme advocated for in [8]. Although the face value of $B_2^{(g)}$ is huge as compared to that of $B_2^{(1)}$ the gluonic distribution amplitude itself is not large as can be seen from Fig. 2, its $x \leftrightarrow 1 - x$ asymmetry and the numerical factors in (2) keep it small. Moreover since it only contributes to NLO its impact on the transition form factors is small resulting in large errors. In order to obtain more precise information on the gluonic distribution amplitude additional constraints from other reactions are required. The inclusive decay $\Upsilon(1S) \rightarrow \eta' X$, discussed in [14], is one such possibility. Others are e.g. $B \rightarrow \pi \eta'$ or $\chi_{cJ} \rightarrow \eta' \eta'$. Finally it is to be emphasized that the approach presented in this article applies to all flavour-neutral pseudoscalar mesons, e.g. for the $\eta(1400)$. The properties of the valence distribution amplitudes (2) make it unlikely that a pseudoscalar meson possesses pure glueball properties. A substantial $q\bar{q}$ Fock component is always there. For flavour-neutral scalar mesons, on the other hand, the situation is different. The properties of the quark and gluon distribution amplitudes are reversed [15]. A strong gg Fock component is therefore not necessarily accompanied by strong $q\bar{q}$ one.

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